Class 7 Solutions Mathematics Chapter 4 Expressions using Letter Numbers

In-Text Questions (Pages 81-83)

Example 1. Shabnam is 3 years older than Aftab. When Aftab's age 10 years, Shabnam's age will be 13 years. Now, Aftab's age is 18 years, what will Shabnam's age be?

Solution: Shabnam's age will be 18 + 3 = 21 years

Use this expression to find A'ftab's age if Shabnam's age is 20.
Solution: According to the expression,
Aftab's age = Shabnam's Age – 3
∴ Aftab's age = 20 – 3 = 17 years

How much should she pay if she buys 8 coconuts and 9 kg jaggery?
Solution: Cost of 8 coconuts = 8 × ₹ 35 = ₹ 280
Cost of 9 kg jaggery = 9 × ₹ 60 = ₹ 540
Total cost = ₹ 280 + ₹ 540 = ₹ 820

Use this expression (or formula) to find the total amount to be paid for 7 coconuts and 4 kg jaggery.
Solution: Expression for total amount is c × 35 + j × 60
Replacing c by 7 and j by 4 in the expression, we get
c × 35 + j × 60 = 7 × 35 + 4 × 60
= 245 + 240
= 485
Hence, the total amount to be paid for 7 coconuts and 4 kg of jaggery is ₹ 485.

What is the perimeter of a square with sidelength 7 cm? Use the expression to find out.
Solution: We have, perimeter of a square of sidelength q = 4 × q
Perimeter of a square of sidelength 7 cm = 4 × 7 cm = 28 cm

Figure it Out (Pages 84-85)

Question 1. Write formulas for the perimeter of:
(a) triangle with all sides equal.
(b) a regular pentagon (as we have learnt last year, we use the word 'regular' to say that all sidelengths and angle measures are equal)
(c) a regular hexagon.
Solution: (a) Let sidelength of each side of triangle = a
So, perimeter of triangle with all sides equal = 3a.

(b) Let sidelength of a regular pentagon = a
So, perimeter of the regular pentagon = 5a

(c) Let sidelength of a regular hexagon = a
So, perimeter of the regular hexagon = 6a

Question 2. Munirathna has a 20 m long pipe. However, he wants a longer watering pipe for his garden. He joins another pipe of some length to this one. Give the expression for the combined length of the pipe. Use the letter-number 'k' to denote the length in meters of the other pipe.
Solution: Length of pipe Munirathna has = 20 m
Length of another pipe he wants to join in meters = k
The combined length of the pipe = (20 + k) m

Question 3. What is the total amount Krithika has, if she has the following number of notes ₹ 100, ₹ 20 and ₹ 5? Complete the following table:

No. of ₹100 notes	No. of ₹20 notes	No. of ₹5 notes	Expression and total amount
3	5	6	
			$6 \times 100 + 4 \times 20 + 3 \times 5 = 695$
8	4	z	
x	y	z	

Solution:

No. of ₹100 notes	No. ₹20 notes	No. ₹5 notes	Expression and total amount
3	5	6	$3 \times 100 + 5 \times 20 + 6 \times 5 = 430$
6	4	3	$6 \times 100 + 4 \times 20 + 3 \times 5 = 695$
8	4	z	$8 \times 100 + 4 \times 20 + z \times 5 = 880 + 5z$
x	y	z	$x \times 100 + y \times 20 + z \times 5 = 100x + 20y + 5z$

Question 4. Venkatalakshmi owns a flour mill. It takes 10 seconds for the roller mill to start running. Once it is running, each kg of grain takes 8 seconds to grind into powder. Which of the expressions below describes the time taken to complete grind 'y' kg of grain, assuming the machine is off initially?
(a) 10 + 8 + y
(b) (10 + 8) × y
(c) 10 × 8 × y
(d) 10 + 8 × y
(e) 10 × y + 8

Solution: Time taken by the roller mill to start running = 10 seconds
Time taken by the roller mill to grind each kg of grain to powder = 8 seconds
Therefore, expression for the time taken to complete grind y kg of grain = 10 + 8 × y
Thus, expression (d) 10 + 8 × y describe the given condition.

Question 5. Write algebraic expressions using letters of your choice.
(a) 5 more than a number
(b) 4 less than a number
(c) 2 less than 13 times a number
(d) 13 less than 2 times a number
Solution: Let letter 'n' represents the number, then
(a) n + 5
(b) n – 4
(c) 13 × n – 2
(d) 2 × n – 13

Question 6. Describe situations corresponding to the following algebraic expressions:
(a) 8 × x + 3 × y
(b) 15 × j – 2 × k
Solution: (a) Kritika bought x number of notebooks andy number of pencils. What is the. total amount she has to pay to the shopkeeper, if the cost of each notebook is ₹ 8 and cost of each pencil is ₹ 3?

(b) Rohan earns ₹ 15 per glass by selling lemonade in his school fete. To maintain the cleanliness he further announced ₹ 2 discount on each glass, the customer return to him. If he sold j number of glasses and k number of glasses were returned to him, what is the total earnings at the end of the day?

Question 7. In a calendar month, if any 2 × 3 grid full of dates is chosen as shown in the picture, expression for the dates in the blank cells if the bottom middle cell has date 'w'.

November 2024

Mon	Tue	Wed	Thu	Fri	Sat	Sun
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

w – 1	w	

Solution:

w – 8	w – 7	w – 6
w – 1	w	w + 1

4.2 Revisiting Arithmetic Expressions
In-Text Questions (Pages 85-86)

We learnt to write expressions as sums of terms, and it became easy for us to read arithmetic expressions. Many times, they could have been read in multiple ways, and it was confusing. We used swapping (adding two numbers in any order) and grouping (adding numbers by grouping them conveniently) to find easy ways of evaluating expressions. Swapping and grouping terms does not change the value of the expression. We also learnt to use brackets in expressions, including brackets with a negative sign outside. We learnt the distributive property (multiple of a sum is the same as sum of multiples).

Now, find the values of the other arithmetic expressions.

1. $23 - 10 \times 2$
2. $83 + 28 - 13 + 32$
3. $34 - 14 + 20$
4. $42 + 15 - (8 - 7)$
5. $68 - (18 + 13)$
6. $7 \times 4 + 9 \times 6$
7. $20 + 8 \times (16 - 6)$

Solution:

3. $34 - 14 + 20$

$34 - 14 + 20 = \boxed{34-14} + \boxed{20} = \boxed{20} + \boxed{20} = \boxed{40}$

4. $42 + 15 - (8 - 7)$

$42 + 15 - (8 - 7) = \boxed{42} + \boxed{15} + \boxed{-1} = \boxed{57} + \boxed{-1} = \boxed{56}$

6. $7 \times 4 + 9 \times 6$

$7 \times 4 + 9 \times 6 = \boxed{7 \times 4} + \boxed{9 \times 6} = \boxed{28} + \boxed{54} = \boxed{82}$

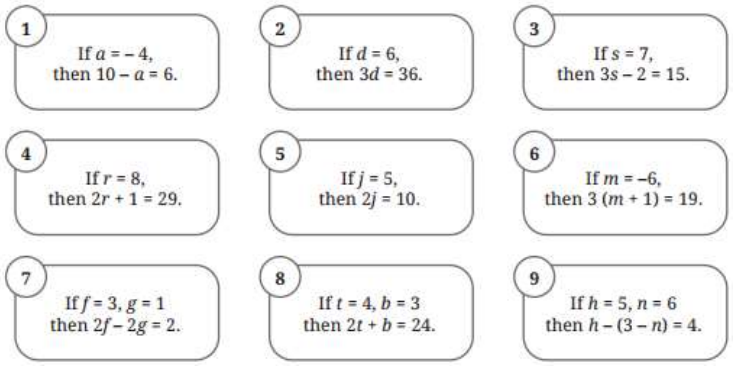
7. $20 + 8 \times (16 - 6)$

$20 + 8 \times (16 - 6) = \boxed{20} + \boxed{8 \times (16 - 6)} = \boxed{20} + \boxed{8 \times 10}$

$= \boxed{20} + \boxed{80} = \boxed{100}$

In-Text Questions (Page 87)

Below are some simplifications, where the letter numbers are replaced by numbers, and the value of the expression is obtained.
1. Observe each of them and identify if there is a mistake.
2. If you think there is a mistake, try to explain what might have gone wrong.
3. Then, correct it and give the value of the expression.



Solution: 1. If a = -4, then 10 – a = 6 is wrong.
As 10 – a = 10 – (-4) = 10 + 4 = 14
So, if a = -4, then the correct expression is 10 – a = 14.

2. If d = 6, then 3d = 36 is wrong.
As, 3d = 3 × 6 = 18
So, if d = 6, then the correct expression is 3d = 18.

3. If s = 7, then 3s – 2 = 15 is wrong.
As 3s – 2 = 3 × 7 – 2 = 21 – 2 = 19
So, if s = 7, then the correct expression is 3s – 2 = 19.

4. If r = 8, then 2r + 1 = 29 is wrong.
As 2r + 1 = 2 × 8 + 1 = 16 + 1 = 17
So, if r = 8, then the correct expression is 2r + 1 = 17.

5. If j = 5, then 2j = 10 is correct.
As 2j = 2 × 5 = 10

6. If m = -6, then 3(m + 1) = 19 is wrong.
As 3(m + 1) = 3 × (-6 + 1) = 3 × (-5) = -15
So, if m = -6, then the correct expression is 3(m + 1) = -15.

7. If f = 3, g = 1, then 2f – 2g = 2 is wrong.
As 2f – 2g = 2 × 3 – 2 × 1 = 6 – 2 = 4
So, if f = 3, g = 1, then the correct expression is 2f – 2g = 4.

8. If t = 4, b = 3, then 2t + b = 24 is wrong.
As 2t + 6 = 2 × 4 + 3 = 8 + 3 = 11
So, if t = 4, b = 3, then the correct expression is 2t + b = 11.

9. If h = 5, n = 6, then h – (3 – n) = 4 is wrong.
As h – (3 – n) = 5 – (3 – 6) = 5 – (-3) = 5 + 3 = 8
So, if h = 5, n = 6, then the correct expression is h – (3 – n) = 8.

4.4 Simplification of Algebraic Expressions
In-Text Questions (Pages 88-89)

Example 5.
Here is a table showing the number of pencils and erasers sold in a shop. The price per pencil is c, and the price per eraser is d. Find the total money earned by the shopkeeper during these three days.

	Day 1	Day 2	Day 3
Pencils (Price '*c*')	5	3	10
Erasers (Price '*d*')	4	6	1

If c = ₹ 50, find the total amount earned by the sale of pencils.

Solution: Total amount earned by the sale of pencils = (5 + 3 + 10) × c
= (5 + 3 + 10) × ₹ 50
= 18 × ₹ 50
= ₹ 900

Write the expression for the total money earned by selling erasers. Then, simplify the expression.
Solution: If the price per eraser is d, then
total money earned by selling erasers = 4 × d + 6 × d + 1 × d
= 4d + 6d + d
= (4 + 6 + 1) × d
= 11 × d
= 11d

Check that both expressions take the same value when c is replaced by different numbers.
Solution: Let c = 5, then
5c + 3c + 10c = 5 × 5 + 3 × 5 + 10 × 5
= 25 + 15 + 50
= 40 + 50
= 90
And 18c = 18 × 5 = 90
Hence, both expressions take the same value when c is replaced by the number 5.
Similarly, we can verify for other numbers.

NCERT In-Text Questions (Pages 91-93)

Could we have written the initial expression as (40x + 75y) + (-6x – 10y)?
Solution: Yes, as (40x + 75y) + (-6x – 10y)

= (40x + 75y) + -(6x + 10y)
= (40x + 75y) – (6x + 10y)

Example 8.
Charu has been through three rounds of a quiz. Her scores in the three rounds are 7p – 3q, 8p – 4q, and 6p – 2q. Here, p represents the score for a correct answer and q represents the penalty for an incorrect answer.
What are her scores in the second and third rounds?

Solution: Her score in the second round = 8p – 4q
= 8 × 4 – 4 × 1 [As p = 4 and q = 1]
= 32 – 4
= 28
Her score in the third round = 6p – 2q
= 6 × 4 – 2 × 1 [As p = 4, q = 1]
= 24 – 2
= 22

What if there is no penalty? What will be the value of q in that situation?
Solution: If there is no penalty, then the value of q will be 0 in that situation.

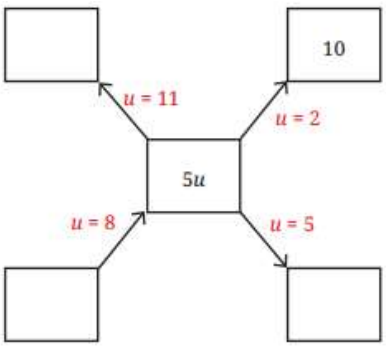
Give some possible scores for Krishita in the three rounds so that they add up to give 23p – 7q.
Solution: Some possible scores of Krishita in three rounds may be 8p – 4q, 9p – 2q, and 6p – q
or 7p – 3q, 10p – 3q, and 6p – q
Here 8p – 4q + 9p – 2q + 6p – q
= (8 + 9 + 6)p – (4 + 2 + 1)q
= 23p – 7q
Also 7p – 3q + 10p – 3q + 6p – q
= (7 + 10 + 6)p – (3 + 3 + 1)q
= 23p – 7q

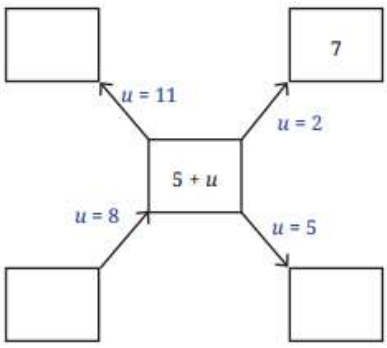
Can we say who scored more? Can you explain why?
How much more has Krishita scored than Charu? This can be found by finding the difference between the two scores.
23p – 7q – (21p – 9q)

Simplify this expression further.
Solution: 23p – 7q – (21p – 9q)
= 23p – 7q – + 21p – 9q
= 23p – 21p – 7q + 9q
= 23p – 21p + 9q – 7q
= (23 – 21)p + (9 – 7)q

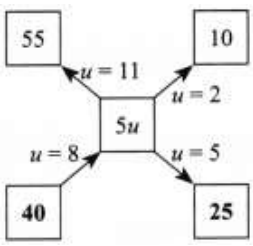
= 2p + 2q
= 2 × (p + q)
= 2(p + q)
The value of 2p + 2q is always positive as long as both p > 0 and q > 0 (since marks and penalties are positive). So, Krishita scored 2(p + q) marks more than Charu.

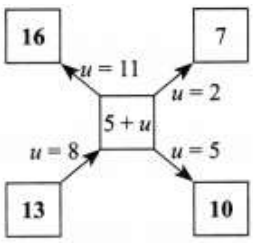
Fill in the blanks below by replacing the letter-numbers with numbers; an example is shown. Then compare the values that 5u and 5 + u take.





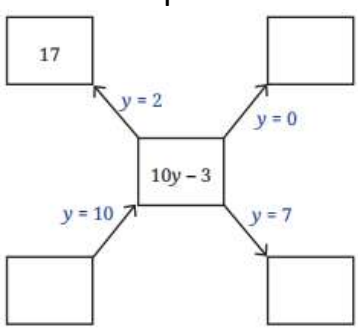
Solution:

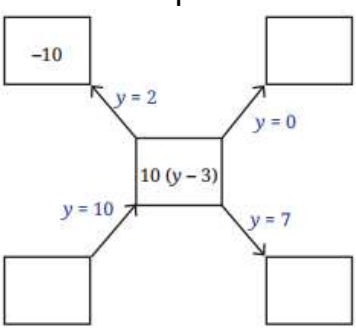




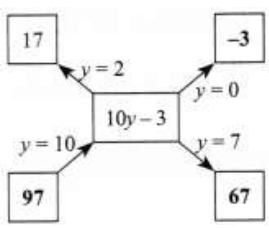
We see that the values of 5u and 5 + u are not equal for different values of u. So, the expressions 5u and 5 + u are not equal.

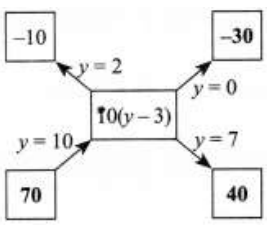
Are the expressions 10y – 3 and 10(y – 3) equal?
10y – 3, short for 10 × y – 3, means 3 less than 10 times y,
10(y – 3), short for 10 × (y – 3), means 10 times (3 less than y).
Let us compare the values that these expressions take for different values of y.





Solution:

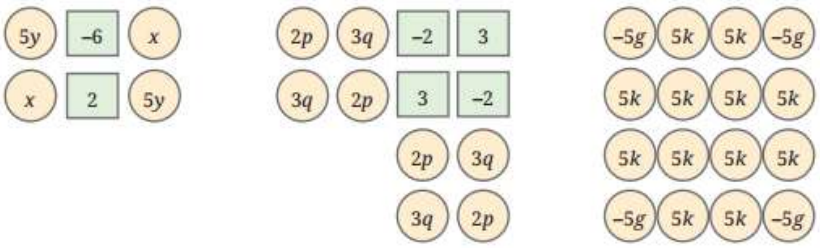




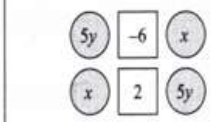
After filling in the two diagrams, do you think the two expressions are equal?
Solution: We see that the values of 10y – 3 and 10(y – 3) are not equal for different values of y. So, the expressions 10y – 3 and 10(y – 3) are not equal.

Figure it Out (Pages 93-94)

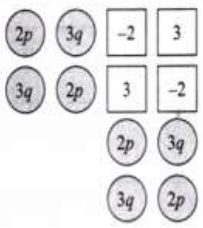
Question 1. Add the numbers in each picture below. Write their corresponding expressions and simplify them. Try adding the numbers in each picture in a couple of different ways and see that you get the same thing.



Solution:



Way I	Way II	Way III
$5y + -6 + x + x + 2 + 5y$ $= 5y + 5y + x + x + -6 + 2$ $= 10y + 2x - 4$	$(2 \times 5y) + (2 \times x) + (-6 + 2)$ $= 2(5y) + (2x) + (-4)$ $= 10y + 2x - 4$	$2 \times (5y + x) + (-6 + 2)$ $= 10y + 2x - 4$



Way I	Way II	Way III
$2p + 3q + -2 + 3 + 3q + 2p + 3 + -2 + 2p + 3q + 3q + 2p$ $= 2p + 2p + 2p + 2p + 3q + 3q + 3q + 3q + -2 + -2 + 3 + 3$ $= 8p + 12q + 2$	$4 \times (2p) + 4 \times (3q) + 2 \times (-2) + 2 \times (3)$ $= 8p + 12q - 4 + 6$ $= 8p + 12q + 2$	$4 \times (2p + 3q) + 2 \times (-2 + 3)$ $= 8p + 12q + 2$

Way I	Way II	Way III
$-5g + 5k + 5k + -5g + 5k + 5k + 5k + 5k + 5k + 5k + 5k + -5g + 5k + 5k + -5g$ $= -5g + -5g + -5g + -5g + 5k + 5k + 5k + 5k + 5k + 5k + 5k + 5k + 5k + 5k + 5k + 5k$ $= -20g + 60k$	$4 \times (-5g) + 12 \times (5k)$ $= -4 \times 5g + 12 \times 5k$ $= -20g + 60k$	$2 \times (-5g) + 2 \times (5k) + 8 \times (5k) + 2 \times (-5g) + 2 \times (5k)$ $= 4 \times (-5g) + 12 \times (5k)$ $= -20g + 60k$

Hence, we see that while we add the expressions in different ways but the sums are always the same.

Question 2. Simplify each of the following expressions:
(a) p + p + p + p, p + p + p + q, p + q + p – q
(b) p – q + p – q, p + q – p + q
(c) p + q – (p + q), p – q – p – q
(d) 2d – d – d – d, 2d – d – d – c
(e) 2d – d – (d – c), 2d – (d – d) – c
(f) 2d – d – c – c

Solution: (a) p + p + p + p = 4p
p + p + p + q = 3p + q
p + q + p – q = 2p

(b) p – q + p – q = 2p – 2q
p + q – p + q = 2q

(c) p + q – (p + q) = 0
p – q – p – q = -2q

(d) 2d – d – d – d = -d
2d – d – d – c = -c

(e) 2d – d – (d – c) = c
2d – (d – d) – c = 2d – c

(f) 2d – d – c – c = d – 2c

NCERT In-Text Questions (Pages 94-95)

Some simplifications of algebraic expressions are done below. The expression on the right-hand side should be in its simplest form.

- Observe each of them and see if there is a mistake.
- If you think there is a mistake, try to explain what might have gone wrong.
- Then, simplify it correctly.

Expression	Simplest Form	Correct Simplest Form
1. $3a + 2b$	5	
2. $3b – 2b – b$	0	
3. $6 (p + 2)$	$6p + 8$	
4. $(4x + 3y) – (3x + 4y)$	$x + y$	
5. $5 – (2 – 6z)$	$3 – 6z$	
6. $2 + (x + 3)$	$2x – 6$	
7. $2y + (3y – 6)$	$– y + 6$	
8. $7p – p + 5q – 2q$	$7p + 3q$	
9. $5 (2w + 3x + 4w)$	$10w + 15x + 20w$	
10. $3j + 6k + 9h + 12$	$3 (j + 2k + 3h + 4)$	
11. $4 (2r + 3s + 5)$	$– 20 – 8r – 12s$	

Solution:

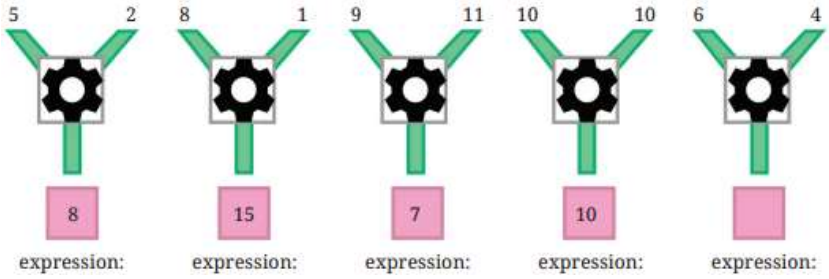
	Expression	Simplest Form	Correct/Wrong	Correct Simplest form
1.	$3a + 2b$	5	Wrong. The letter-numbers of the two terms are different, so they cannot be added.	It is in simplest form itself. $3a + 2b$
2.	$3b - 2b - b$	0	Correct. As, $3b - 2b - b = 3b - 3b = 0$	
3.	$6(p + 2)$	$6p + 8$	Wrong. As, $6(p + 2) = 6 \times p + 6 \times 2$ $= 6p + 12$	$6p + 12$
4.	$(4x + 3y) - (3x + 4y)$	$x + y$	Wrong. As, $(4x + 3y) - (3x + 4y)$ $= 4x + 3y - + 3x - + 4y$ $= 4x - + 3x + 3y - + 4y$ $= (4x - 3x) + (3y - 4y) = x - y$ So, the + sign is wrong	$x - y$
5.	$5 - (2 - 6z)$	$3 - 6z$	Wrong. As, $5 - (2 - 6z) = 5 - 2 + 6z$ $= 3 + 6z$ So, instead of '–' sign '+' should be there.	$3 + 6z$
6.	$2 + (x + 3)$	$2x - 6$	Wrong. As, $2 + (x + 3) = 2 + x + 3$ $= 2 + 3 + x = 5 + x$	$5 + x$
7.	$2y + (3y - 6)$	$-y + 6$	Wrong. As, $2y + (3y - 6) = 2y + 3y + -6$ $= 5y - 6$	$5y - 6$
8.	$7p - p + 5q - 2q$	$7p + 3q$	Wrong. As, $7p - p + 5q - 2q = 6p + 3q$ So, instead of 7, 6 should be there.	$6p + 3q$
9.	$5(2w + 3x + 4w)$	$10w + 15x + 20w$	Wrong. As, $5(2w + 3x + 4w)$ can be further simplified as $10w + 15x + 20w = 10w + 20w + 15x = 30w + 15x$	$30w + 15x$
10.	$3j + 6k + 9h + 12$	$3(j + 2k + 3h + 4)$	Correct.	The given expression is already in simplest form. $3j + 6k + 9h + 12$
11.	$4(2r + 3s + 5)$	$-20 - 8r - 12s$	Wrong. As, $4(2r + 3s + 5)$ $= 4 \times 2r + 4 \times 3s + 4 \times 5$ $= 8r + 12s + 20$ Instead of '–' signs '+' signs should be there.	$8r + 12s + 20$

Take a look at all the corrected simplest forms (i.e., brackets are removed, like terms are added, and terms with only numbers are also added). Is there any relation between the number of terms and the number of letter-numbers these expressions have?
Solution: Yes

- If the expression contains a term having only a number, the number of terms = number of letter-numbers + 1
- If an expression has no term that has only numbers, then number of terms = number of letter-numbers

4.5 Pick Patterns and Reveal Relationships
Formula Detective
NCERT In-Text Questions (Pages 95-96)

Find out the formula of this number machine.

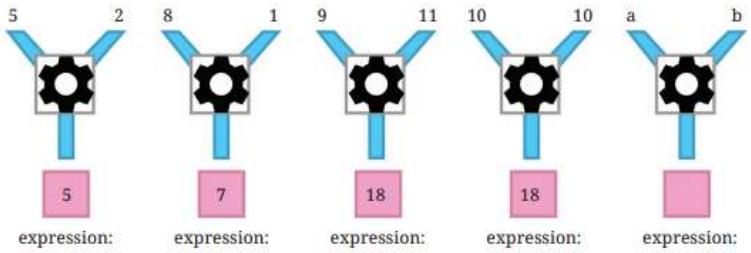


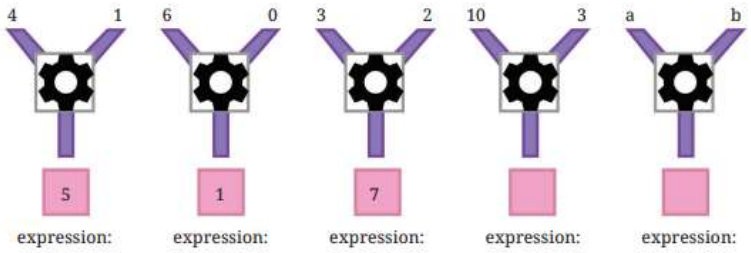
The formula for the number machine above is "two times the first number minus the second number". When written as an algebraic expression, the formula is 2a-b. The expression for the first set of inputs is 2 × 5 – 2 = 8. Check that the formula holds for each set of inputs.

Solution: Yes, the formula holds for each set of inputs.

As, 2 × 8 – 1 = 15; 2 × 9 – 11 = 7; 2 × 10 – 10 = 10; and 2 × 6 – 4 = 8

Find the formulas of the number machines below and write the expression for each set of inputs.





Solution: The formula for the number machines in the first row is "sum of first number and second number minus two," and the expression is a + b – 2.

The expression for each set of inputs is:

5 + 2 – 2 = 5, 8 + 1 – 2 = 7, 9 + 11 – 2 = 18, 10 + 10 – 2 = 18, and a + b – 2

The formula for the number machines in the second row is "product of first number and second number plus one," and the expression is a × b + 1.

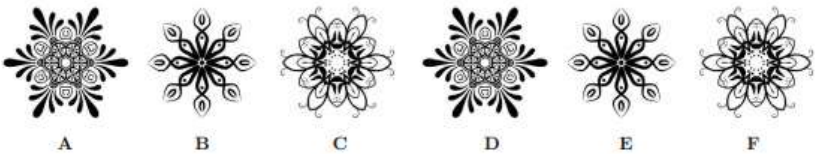
The expression for each set of inputs is:

4 × 1 + 1 = 5, 6 × 0 + 1 = 1, 3 × 2 + 1 = 7, 10 × 3 + 1 = 31, and a × b + 1 = ab + 1.

Now, make a formula on your own. Write a few number machines as examples using that formula. Challenge your classmates to figure it out!
Solution: Do it yourself.

Algebraic Expressions to Describe Patterns
In-Text Questions (Pages 96-97)

Example 12. Somjit noticed a repeating pattern along the border of a saree.



Use this to find what design appears at positions 99, 122, and 148.
Solution:

- For 99, the remainder on division by 3 is 0, i.e., it is a multiple of 3. So, at position 99, design C will appear.
- For 122, the remainder on division by 3 is 2, i.e., it is 1 less than a multiple of 3, i.e., 3n – 1. So, at position 122, design B will appear.
- For 148, the remainder on division by 3 is 1, i.e., it is 2 less than a multiple of 3, i.e., 3n – 2. So, at position 148, design A will appear.

Patterns in a Calendar

In-Text Questions (Page 99)

Verify this expression for diagonal sums by considering any 2 × 2 square and taking its top left number to be 'a'.
Solution: Let a = 2, then

a	$a+1$
$a+7$	$a+8$

=

2	3
9	10

Here, the diagonal sums are 2 + 10 = 12 and 9 + 3 = 12
And 2a + 8 = 2 × 2 + 8 = 12
Hence, the diagonal sum is equal to 2a + 8.

Find the sum of all the numbers. Compare it with the number in the centre: 15. Repeat this for another set of numbers that form this shape. What do you observe?
Solution: Sum of all numbers = 8 + 14 + 15 + 16 + 22 = 75
The sum is 5 times the number in the centre.
Now, let the number at the centre: 20, then the shape is

	13	
19	20	21
	27	

Sum of all the numbers = 13 + 19 + 20 + 21 + 27 = 100 = 20 × 5

Further, let the number at the centre: 12, then the shape is

	5	
11	12	13
	19	

Sum of all numbers = 5 + 11 + 12 + 13 + 19 = 60 = 12 × 5
Hence, we see that the total sum is always 5 times the number in the centre.

Will this always happen? How do you show this?
Solution: The general formula for a 3 × 3 square with centre number a is shown here,

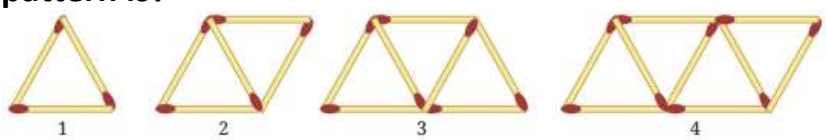
	$a-7$	
$a-1$	a	$a+1$
	$a+7$	

Here, the sum is a – 7 + a – 1 + a + a + 1 + a + 7 = a + a + a + a + a – 7 – 1 + 1 + 7
So, the sum of numbers = 5a (5 times the number in the centre).

Find other shapes for which the sum of the numbers within the figure is always a multiple of one of the numbers.
Solution: Do it yourself.

Matchstick Patterns

Look at the picture below. It is a pattern using matchsticks. Can you identify what the pattern is?



We can see that Step 1 has 1 triangle, Step 2 has 2 triangles, Step 3 has 3 triangles, and so on.

In-Text Questions (Page 101)

What are these numbers in Step 3 and Step 4?
Solution:
In step 3, there are 3 matchsticks placed horizontally and 4 matchsticks placed diagonally.
In step 4, there are 4 matchsticks placed horizontally and 5 matchsticks placed diagonally.

How does the number of matchsticks change in each orientation as the steps increase? Write an expression for the number of matchsticks at Step 'y in each orientation. Do the two expressions add up to 2y + 1?
Solution: The pattern in the horizontally placed matchsticks is: 1, 2, 3, 4,….
In general, n for the nth step.
The pattern in the diagonally placed matchsticks is: 2, 3, 4, 5,….

In general, n + 1 for the nth step.
∴ The number of matchsticks placed horizontally at step 'y' is y.
The number of matchsticks placed diagonally at step 'y' is y + 1.
Now, these two expressions add up to y + y + 1 = 2y + 1.

Figure it Out (Pages 102-105)

For the problems asking you to find suitable expression(s), first try to understand the relationship between the different quantities in the situation described. If required, assume some values for the unknowns and try to find the relationship.

Question 1.
One plate of Jowar roti costs ₹ 30, and one plate of Pulao costs ₹ 20. If x plates of Jowar roti and y plates of pulao were ordered in a day, which expression(s) describe the total amount in rupees earned that day?
(a) 30x + 20y
(b) (30 + 20) × (x + y)
(c) 20x + 30y
(d) (30 + 20) × x + y
(e) 30x – 20y

Solution: (a) Cost of one plate of Jowar roti = ₹ 30
∴ Cost of x plate of Jowar roti = 30x
Cost of one plate of Pulao = ₹ 20
∴ Cost of y plate of Pulao = 20y
So, the expression for the total amount earned that day = 30x + 20y

Question 2. Pushpita sells two types of flowers on Independence Day: champak and marigold. 'p' customers only bought champak, 'q' customers only bought marigold, and 'r' customers bought both. On the same day, she gave away a tiny national flag to every customer. How many flags did she give away that day?
(a) p + q + r
(b) p + q + 2r
(c) 2 × (p + q + r)
(d) p + q + r + 2
(e) p + q + r + 1
(f) 2 × (p + q)

Solution: (a) Number of customers who bought only champak = p
Number of customers who bought only marigold = q
Number of customers who bought both = r
As Pushpita gave away a tiny national flag to every customer.
So, the number of flags she gives away that day = p + q + r.

Question 3. A snail is trying to climb up the wall of a deep well. During the day it climbs up 'u' cm and during the night it slowly slips down 'd' cm. This happens for 10 days and 10 nights.
(a) Write an expression describing how far away the snail is from its starting position.
(b) What can we say about the snail's movement if d > u?

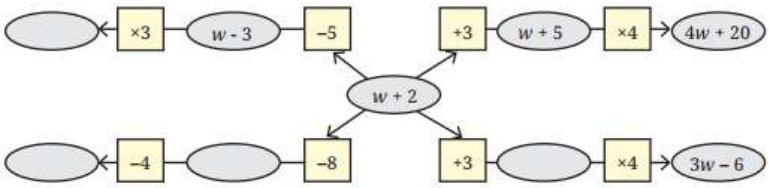
Solution: (a) During the day, the snail climbs up 'u' cm.
During the night snail slips down d cm.
So, the net distance covered in one day is u-d.
So, in 10 days and 10 nights, the net distance covered by the snail = 10(u-d).
Hence, the expression describing how far away the snail is from it starting position is 10(u – d) cm.

(b) If d > u, the snail slips down more than it climbs.
It means the snail will never reach the top.

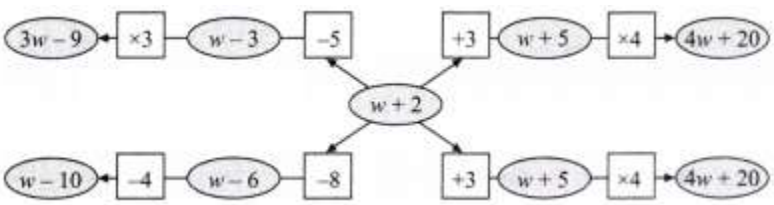
Question 4. Radha is preparing for a cycling race and practices daily. The first week, she cycles 5 km every day. Every week, she increases the daily distance cycled by 'z' km. How many kilometers would Radha have cycled after 3 weeks?

Solution: In the first week, Radha cycled 5 km every day.
So, she cycled 5 × 7 = 35 km in the first week.
In the second week, Radha cycles (5 + z) km every day.
So, she cycled (5 + z) × 7 = (35 + 7z) km in second week.
In the third week, she cycles (5 + z + z = 5 + 2z) km every day.
So, she cycled (5 + 2z) × 7 = (35 + 14z) km in third week.
So, number of kilometres Radha cycled in 3 weeks = 35 + (35 + 7z) + (35 + 14z)
= (35 + 35 + 35) + (7z + 14z)
= 105 + 21z km

Question 5. In the following figure, observe how the expression w + 2 becomes 4w + 20 along one path. Fill in the missing blanks on the remaining paths. The ovals contain expressions, and the boxes contain operations.

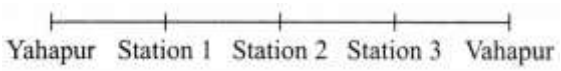


Solution:



Question 6. A local train from Yahapur to Vahapur stops at three stations at equal distances along the way. The time taken in minutes to travel from one station to the next station is the same and is denoted by t. The train stops for 2 minutes at each of the three stations.
(a) If t = 4, what is the time taken to travel from Yahapur to Vahapur?
(b) What is the algebraic expression for the time taken to travel from Yahapur to Vahapur?
[Hint: Draw a rough diagram to visualise the situation]

Solution: (a) The train from Yahapur to Vahapur stops at 3 stations, and stops for 2 minutes at every station.
Time taken in travelling = 4t
At t = 4, time taken in travelling = 4 × 4 = 16 minutes
Time taken during stoppages = 3 × 2 = 6 minutes
So, the time taken to travel from Yahapur to Vahapur = 16 + 6 = 22 minutes.

(b) 

Let the time taken to travel from one station to another station = t
So, time taken to travel from Yahanpur to Vahapur = 4t
As there are three stoppages between these two stations, and the train stops for 2 minutes at each stoppage,
therefore total time taken during stoppages = 2 × 3 = 6 minutes
So, the algebraic expression for total time taken is 4t + 6.

Question 7. Simplify the following expressions:
(a) 3a + 9b – 6 + 8a – 4b – 7a + 16
(b) 3(3a – 3b) – 8a – 4b – 16
(c) 2(2x – 3) + 8x + 12
(d) 8x – (2x – 3) + 12
(e) 8h – (5 + 7h) + 9
(f) 23 + 4(6m – 3n) – 8n – 3m – 18

Solution: (a) 3a + 9b – 6 + 8a – 4b – 7a + 16
= (3a + 8a – 7a) + (9b – 4b) + (-6 + 16)
= 4a + 5b + 10

(b) 3(3a – 3b) – 8a – 4b – 16
= 9a – 9b – 8a – 4b – 16
= (9a – 8a) + (-9b – 4b) – 16
= a – 13b – 16

(c) 2(2x – 3) + 8x + 12
= 4x – 6 + 8x + 12

= (4x + 8x) + (-6 + 12)
= 12x + 6

(d) 8x – (2x – 3) + 12
= 8x – 2x + 3 + 12
= 6x + 15

(e) 8h – (5 + 7h) + 9
= 8b – 5 – 7b + 9
= (8b – 7b) + (-5 + 9)
= b + 4

(f) 23 + 4(6m – 3n) – 8n – 3m – 18
= 23 + 24m – 12n – 8n – 3m – 18
= (23 – 18) + (24m – 3m) + (-12n – 8n)
= 5 + 21m – 20n

Question 8. Add the expressions given below:
(a) 4d – 7c + 9 and 8c – 11 + 9d
(b) -6f + 19 – 8s and -23 + 13f + 12s
(c) 8d – 14c + 9 and 16c – (11 + 9d)
(d) 6f – 20 + 8s and 23 – 13f – 12s
(e) 13m – 12n and 12n – 13m
(f) -26m + 24n and 26m – 24n

Solution: (a) 4d – 7c + 9 and 8c – 11 + 9d
= 4d – 7c + 9 + 8c – 11 + 9d
= (4d + 9d) + (-7c + 8c) + (9 – 11)
= 13d + c – 2

(b) -6f + 19 – 8s and -23 + 13f + 12s
= -6f + 19 – 8s + -23 + 13f + 12s
= (-6f + 13f) + (-8s + 12s) + (19 – 23)
= 7f + 4s – 4

(c) 8d – 14c + 9 and 16c – (11 + 9d)
= 8d – 14c + 9 + 16c – 11 – 9d
= 8d – 9d – 14c + 16c + 9 – 11
= -d + 2c – 2
= 2c – d – 2

(d) 6f – 20 + 8s and 23 – 13f – 12s
= 6f – 20 + 85 + 23 – 13f – 125

= (6f – 13f) + (8s – 12s) + (-20 + 23)
= -7f – 4s + 3

(e) 13m – 12n and 12n – 13m
= 13m – 12n + 12n – 13m
= (13m – 13m) + (-12n + 12n)
= 0

(f) -26m + 24n and 26m – 24n
= -26m + 24n + 26m – 24n
= (-26m + 26m) + (24n – 24n)
= 0

Question 9. Subtract the expressions given below:
(a) 9a – 6b + 14 from 6a + 9b – 18
(b) -15x + 13 – 9y from 7y – 10 + 3x
(c) 17g + 9 – 7h from 11 – 10g + 3h
(d) 9a – 6b + 14 from 6a – (9b + 18)
(e) 10x + 2 + 10y from -3y + 8 – 3x
(f) 8g + 4h – 10 from 7h – 8g + 20

Solution: (a) (6a + 9b – 18) – (9a – 6b + 14)
= 6a + 9b – 18 – 9a + 6b – 14
= (6a – 9a) + (9b + 6b) + (-18 – 14)
= -3a + 15b – 32

(b) (7y – 10 + 3x) – (-15x + 13 – 9y)
= 7y – 10 + 3x + 15x – 13 + 9y
= (7y + 9y) + (3x + 15x) + (-10 – 13)
= 16y + 18x – 23

(c) (11 – 10g + 3h) – (17g + 9 – 7h)
= 11 – 10g + 3h – 17g – 9 + 7h
= (11 – 9) + (-10g – 17g) + (3h + 7h)
= 2 – 27g + 10h
= 10h – 27g + 2

(d) 6a – (9b + 18) – (9a – 6b + 14)
= 6a – 9b – 18 – 9a + 6b – 14
= (6a – 9a) + (-9b + 6b) + (-18 – 14)
= -3a – 3b – 32
= -(3a + 3b + 32)

(e) (-3y + 8 – 3x) – (10x + 2 + 10y)
= -3y + 8 – 3x – 10x – 2 – 10y
= (-3y – 10y) + (-3x – 10x) + (8 – 2)
= -13y – 13x + 6

(f) (7h – 8g + 20) – (8g + 4h – 10)
= 7h – 8g + 20 – 8g – 4h + 10
= (7h – 4h) + (-8g – 8g) + (20 + 10)
= 3h – 16g + 30

Question 10. Describe situations corresponding to the following algebraic expressions:
(a) 8x + 3y
(b) 15x – 2x

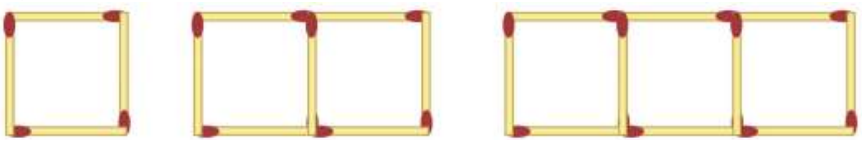
Solution: (a) A notebook costs ₹ 8 and a pen costs ₹ 3. If you buy x notebooks and y pens. Then the total cost bear by you is ₹ (8x + 3y).

(b) A fruit seller has 15 boxes of apples, each box containing x apples. Before selling them, he found that 2 boxes of apples were rotten. The number of fresh apples left is 15x – 2x.

Question 11.
Imagine a straight rope. If it is cut once as shown in the picture, we get 2 pieces. If the rope is folded once and then cut as shown, we get 3 pieces. Observe the pattern and find the number of pieces if the rope is folded 10 times and cut. What is the expression for the number of pieces when the rope is folded r times and cut?

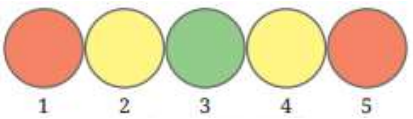
Solution: Step 1 (0 fold): We get 0 + 2 = 2 pieces
Step 2 (1 fold): We get 1 + 2 = 3 pieces
Step 3 (2 folds): We get 2 + 2 = 4 pieces
In the same way, if the rope is folded 10 times and cut, we get 10 + 2 = 12 pieces.
In the same way, when the rope is folded r times and cut, we get r + 2 pieces.

Question 12.
Look at the matchstick pattern below. Observe and identify the pattern. How many matchsticks are required to make 10 such squares? How many are required to make w squares?



Solution: Step 1: To make 1 square, we need 4 matchsticks.
Step 2: To make 2 squares, we need 4 + 3 = 7 matchsticks
Step 3: To make 3 squares, we need 4 + 3 + 3 = 10 matchsticks.
And to make w squares we need = 4 + (w – 1) × 3
= 4 + 3(w – 1)
= (4 + 3w – 3)
= 3w + 1 matchsticks.
To make 10 squares, substitute 10 for w:
3(10) + 1 = 30 + 1 = 31 matchsticks

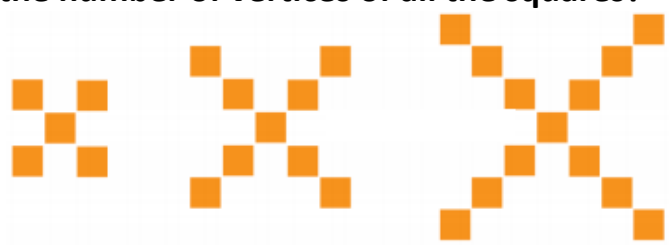
Question 13. Have you noticed how the colours change in a traffic signal? The sequence of colour changes is shown below. Find the colour at positions 90, 190, and 343. Write expressions to describe the positions for each colour.



.....

Solution: The sequence of red light: 1, 5, 9, ...
In general, 4n – 3 positions
The sequence of green light: 3, 7, 11,.....
In general; 4n – 1 positions
The sequence of yellow light: 2, 4, 6, ...
In general, 2n positions
Since 90 and 190 are even numbers, it will be 2n positions.
Now, 343 ÷ 4 = 85 quotient + 3 remainder.
So, it matches a 4n-1 position.
So, colour at positions 90, 190, and 343 are yellow, yellow, and green, respectively.

Question 14. Observe the pattern below. How many squares will be there in Step 4, Step 10, Step 50? Write a general formula. How would the formula change if we want to count the number of vertices of all the squares?



Solution:
Number of squares in step 1 = 5
Number of squares in step 2 = 5 + 4 = 9
Number of squares in step 3 = 5 + 4 + 4 = 5 + 2 × 4 = 13
So, number of squares in step 4 = 5 + 4 + 4 + 4 = 5 + 3 × 4 = 17
So, number of squares in step 10 = 5 + 9 × 4 = 41
And, number of squares in step 50 = 5 + 49 × 4 = 201
So, the general formula = 5 + (n – 1) × 4 = 5 + 4(n – 1) = 5 + 4n – 4 = 4n + 1.

Since 1 square has 4 vertices, the number of vertices (4n + 1) squares have 4(4n + 1) = 16n + 4.

Question 15. Numbers are written in a particular sequence in this endless 4-column grid.

1	2	3	4
1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

(a) Give expressions to generate all the numbers in a given column (1, 2, 3, 4).
(b) In which row and column will the following numbers appear:
(i) 124
(ii) 147
(iii) 201
(c) What number appears in row r and column c?
(d) Observe the positions of multiples of 3.
Do you see any pattern in it? List other patterns that you see.

Solution: (a) Expression to generate all the numbers in a given column (1, 2, 3, 4)
Let r be the row number.
Column 1: 1, 5, 9, 13,…… which starts at 1 and adds 4 each row.
So, number in the rth row of column 1 = 4 × (r – 1) + 1
Column 2: 4 × (r – 1) + 2
Column 3: 4 × (r – 1) + 3
Column 4: 4 × (r – 1) + 4
If c is the column number, then the general formula to generate all numbers is 4 × (r – 1) + c.

(b) (i) We divide each number by 4 to find its row and column
124 ÷ 4 ⇒ Quotient = 31 and remainder is 0
∴ 124 = 4 × 31 + 0 or 4 × 30 + 4
Comparing it with 4 × (r – 1) + c, we get
r – 1 = 30, c = 4
So, r = 31 and c = 4
So, row is 31 and column is 4

(ii) 147 ÷ 4 ⇒ Quotient = 36 and remainder is 3
∴ 147 = 4 × 36 + 3
Comparing it with 4 × (r – 1) + c, we get
r – 1 = 36, c = 3
So, 147 will appear at row 36 + 1 = 37 and column 3

(iii) $201 \div 4 \Rightarrow$ Quotient = 50 and remainder is 1
$\therefore 201 = 4 \times 50 + 1$
Comparing it with $4 \times (r - 1) + c$, we get
$r - 1 = 50, c = 1$
So, 201 will appear at row 51 and column 1.

(c) The number that appears in row r and column c is $4(r - 1) + c$.

(d) Every third number is a multiple of 3.
We can observe that even numbers always appear in column 2 and column 4.
Odd numbers always appear in column 1 and column 3.
Every row has 2 odd and 2 even numbers.
The sum of each row increases by 16.
(e.g., Row 1: 1 + 2 + 3 + 4 = 10, Row 2: 5 + 6 + 7 + 8 = 26, Row 3: 9 + 10 + 11 + 12 = 42)